PRINCIPLES OF GNSS, INERTIAL, AND MULTISENSOR INTEGRATED NAVIGATION SYSTEMS

BY PAUL D. GROVES

List of symbols

Here, the symbols that appear in the book's equations are listed. They are divided into matrices, denoted by upper case bold, vectors, denoted by lower case bold, scalars, denoted by italics, subscripts and superscripts, and qualifiers. Subscripts and superscripts are only listed separately where they are used with more than one parent symbol, otherwise the compound symbol is listed. Components of vectors and matrices are denoted by the equivalent scalar with subscript indices added. The magnitude of a vector is denoted by the equivalent scalar with no subscript index. Sub-matrices retain matrix notation, but have subscript indices added.

Matrices

$\mathbf{A}$	generic matrix
$\mathbf{A}$	smoothing gain
$\mathbf{B}$	generic matrix
$\mathbf{B}$	covariance of the state vector difference
$\mathbf{C}$	co-ordinate transformation matrix
$\mathbf{C}$	generic matrix
$\mathbf{C}^{-}$	covariance of measurement innovations
$\mathbf{C}^{+}$	covariance of measurement residuals
$\mathbf{F}$	system matrix
$\mathbf{G}$	system noise distribution matrix
$\mathbf{G}$	geometry matrix
$\mathbf{G}_g$	gyro g-dependent errors
$\mathbf{H}$	measurement matrix
$\mathbf{I}_n$	$n \times n$ identity matrix (diagonal elements = 1, off-diagonal elements = 0)
$\mathbf{J}$	measurement matrix for unestimated parameters
$\mathbf{K}$	Kalman gain
$\mathbf{M}$	scale factor and cross-coupling errors
$\mathbf{P}$	error covariance matrix
$\mathbf{P}$	distribution covariance matrix
$\mathbf{Q}$	system noise covariance matrix
$\mathbf{Q}_U$	system noise covariance matrix for unestimated parameters
$\mathbf{R}$	measurement noise covariance matrix
$\mathbf{T}$	position change transformation matrix
$\mathbf{U}$	correlation matrix between states and unestimated parameters
$\mathbf{W}$	error covariance matrix for unestimated parameters
$\mathbf{\Phi}$	transition matrix
$\mathbf{\Phi}_U$	transition matrix for unestimated parameters
$\mathbf{\Psi}$	transition matrix linking states with unestimated parameters
$\mathbf{\Omega}$	skew-symmetric matrix of angular rate

Vectors

$\mathbf{a}$	acceleration
$\mathbf{a}$	generic vector
$\mathbf{b}$	bias errors

$\mathbf{b}$	generic vector
$\mathbf{c}$	step length estimation coefficients
$\mathbf{c}$	generic vector
$\mathbf{c}_i$	i^{th} row of co-ordinate transformation matrix
$\mathbf{d}$	generic vector
$\mathbf{f}$	specific force
$\mathbf{f}$	system function
$\mathbf{g}$	acceleration due to gravity
$\mathbf{g}$	generic function
$\mathbf{h}$	measurement function
$\mathbf{h}$	angular momentum
$\mathbf{k}_n$	Runge-Kutta integration intermediate step result
$\mathbf{l}$	lever arm
$\mathbf{m}$	cross-coupling errors
$\mathbf{m}$	magnetic flux density
$\mathbf{m}$	quantities measured
$\mathbf{p}$	curvilinear position (geodetic latitude, longitude and geodetic height)
$\mathbf{p}$	parity vector
$\mathbf{q}$	quaternion attitude
$\mathbf{r}$	Cartesian position
$\mathbf{s}$	scale factor errors
$\mathbf{s}_{cg}$	receiver clock g-dependent error coefficients
$\mathbf{u}$	unit vector and line of sight unit vector
$\mathbf{u}$	control vector
$\mathbf{v}$	velocity
$\mathbf{w}$	vector of white noise sources
$\mathbf{w}_m$	measurement noise vector
$\mathbf{w}_s$	system noise vector
$\mathbf{x}$	generic vector or set of observations
$\mathbf{x}$	state vector
$\mathbf{y}^-$	normalized measurement innovation vector
$\mathbf{y}^+$	normalized measurement residual vector
$\mathbf{z}$	measurement vector
$\boldsymbol{\alpha}$	attitude increment
$\boldsymbol{\gamma}$	acceleration due to the gravitational force
$\Delta\mathbf{r}$	position displacement
$\delta\mathbf{x}$	state vector residual
$\delta\mathbf{z}^-$	measurement innovation
$\delta\mathbf{z}^+$	measurement residual
$\boldsymbol{\eta}$	flexure coefficients
$\boldsymbol{\mu}$	means
$\boldsymbol{\rho}$	rotation vector
$\boldsymbol{\tau}$	torque
$\boldsymbol{\upsilon}$	integrated specific force
$\boldsymbol{\psi}$	Euler attitude {roll, pitch, yaw} (no superscript)
$\boldsymbol{\psi}$	Small-angle attitude (superscript indicates resolving axes)
$\boldsymbol{\omega}$	angular rate

Scalars

A	area
a	length of the semi-major axis
a	integer ambiguity
a	generic scalar
A_a	signal amplitude following amplification
a_f	satellite clock calibration coefficient
B	magnetic flux density
b	bias error
b	generic scalar
B_{L_CA}	carrier-phase tracking-loop bandwidth
B_{L_CF}	carrier-frequency tracking-loop bandwidth
B_{L_CO}	code tracking-loop bandwidth
B_{PC}	double-sided pre-correlation bandwidth
C	spreading code
C	orbital harmonic correction term
c	speed of light in free space or fiber-optic coil
c	magnetic compass calibration coefficient
c	generic scalar
C/N_0	$10\log_{10}$ carrier power to noise density
c/n_0	carrier power to noise density
D	navigation data message
D	dilution of precision
D	code discriminator function
d	spacing of early and late correlation channels in code chips
d	depth
d	generic scalar
E	eccentric anomaly
e	eccentricity of the ellipsoid
e_o	eccentricity of the orbit
F	carrier-frequency discriminator function
F	cumulative probability
f	flattening of the ellipsoid
f	frequency
f	probability density function
f_a	ADC sampling frequency
H	orthometric height
h	geodetic height
h_i	mean ionosphere height
h	scaling factor in measurement matrix
I	intensity
i	inclination angle
J_2	Earth’s second gravitational constant
k	discriminator gain
k_T	atmospheric temperature gradient
K	loop gain
L	geodetic latitude

l	number of system-noise-vector components
l	number of filter hypotheses
l	number of matrix rows
M	mean anomaly
M	narrow-band to wide-band accumulation interval
m	number of measurement-vector components
m	number of smoothing iterations
m	quantity measured
m	number of vector components or matrix rows/columns
N	geoid height
N	number of turns
N	normalization function
N	number of samples
N	sample from Gaussian distribution
n	root power spectral density
n	number of state-vector components
n	number of vector components or matrix columns
n	number of observations
n	number of degrees of freedom of chi-square distribution
n_k	number of measurement-vector hypotheses at iteration k
n_{rcd}	root power spectral density of receiver clock drift
n_0	noise power spectral density (not root)
P	power
p	first component of angular-rate vector
p	pressure
p	probability
q	second component of angular-rate vector
R_0	equatorial Earth radius
R	correlation function
R	gas constant
r	third component of angular-rate vector
r	iteration counter in summation
r	root mean square
R	average Earth radius
R_E	transverse radius of curvature
R_N	meridian radius of curvature
R_P	polar Earth radius
S	sub-carrier function
s	signal amplitude
s	root chi-square test statistic
s	scale factor error
T	test statistic
T	temperature
T	track width
$T_{b\mu}$	innovation-bias threshold
t	time
t_{oe}	reference time of ephemeris
t_{sa}	time of signal arrival

t_{st}	time of signal transmission
t'_{st}	code phase
u	corrected argument of latitude
W	weighting factor
w	white noise source
x	generic process
x	first component of Cartesian position or a generic vector
x	code tracking error in code chips
x	generic argument of probability density
$x_{-/+}$	confidence limits
y	second component of Cartesian position or a generic vector
z	third component of Cartesian position or a generic vector
Z_{cc}	correlator-comparison measurement
α	relative amplitude of multipath component
α	magnetic declination angle/variation
β	magnitude of the projection of position onto the equatorial plane
Γ	gamma function
γ	magnetic inclination/dip angle
Δ	range lag of multipath component
δ	range lag of multipath component in code chips
δ	Kronecker delta function (equals one when indices match and zero otherwise)
Δf	Doppler frequency shift
Δ_{ij}	scalar product of i^{th} and j^{th} co-ordinate transformation matrix rows
Δn	mean motion difference from computed value
Δr	distance travelled
Δx	rise time of signal waveform in code chips
$\Delta\rho_{dc}$	differential correction
$\delta\rho_e$	range error due to ephemeris data
$\Delta\rho_{ic}$	ionosphere correction
$\delta\rho_i$	ionosphere propagation error
$\delta\rho_{ie}$	Sagnac correction
$\delta\rho_m$	range error due to multipath
$\Delta\rho_R$	differenced pseudo-range
$\Delta\rho_{rc}$	relative receiver clock offset
$\delta\rho_{rc}$	receiver clock offset
$\Delta\rho_{r\phi}$	relative receiver phase offset
$\Delta\rho_{sc}$	satellite clock correction
$\delta\rho_s$	range error due to satellite clock error
$\Delta\rho_{tc}$	troposphere correction
$\delta\rho_t$	troposphere propagation error
$\delta\rho_w$	pseudo-range tracking error
$\delta\rho_\varepsilon$	measurement residual of single-point navigation solution
θ	pitch or elevation angle
θ	generic angle
θ_{nu}	elevation angle of satellite line of sight vector
λ	longitude

Λ	likelihood
λ_{ca}	carrier wavelength
λ_0	wavelength
μ	resultant angle
μ	Earth's gravitational constant
μ	mean innovation test statistic
μ	mean
ν	true anomaly
ρ	range or pseudo-range
ρ	density
ρ_C	corrected pseudo-range measured by user equipment
ρ_R	pseudo-range measured by user equipment
ρ_T	true range
σ	standard deviation or error standard deviation
σ_{IQ}	noise standard deviation of accumulated correlator outputs
τ	correlation time
τ	propagation time
τ_a	correlator accumulation interval
τ_i	inertial navigation integration interval
τ_o	odometer measurement interval
τ_P	PDR measurement interval
τ_s	system propagation time
Φ	geocentric latitude
Φ	argument of latitude
Φ	carrier-phase discriminator function
ϕ	roll or bank angle
ϕ	phase
χ^2	chi-square statistic
ψ	yaw or heading angle
ψ_{bh}	boresight angle
ψ_{nu}	azimuth angle of satellite line of sight vector
Ω	longitude/ right ascension of the ascending node
ω	angular frequency
ω	argument of perigree

Subscripts and superscripts

A	denotes local magnetic anomalies
A	denotes accelerometer indicated
A	denotes attitude-matching transfer alignment measurement
a	denotes a vibrating element
a	denotes accelerometer
a	denotes user antenna body co-ordinate frame
a	denotes at the antenna
ASF	denotes additional secondary factor
B	denotes barometric height measurement
b	denotes body or INS body co-ordinate frame

b	denotes backwards filter
b	denotes barometric altimeter
bad	denotes accelerometer dynamic bias
bgd	denotes gyro dynamic bias
C	denotes receiver-generated carrier
C	denotes post-correlation
c	denotes from the coil
c	denotes due to or of coning motion
c	denotes cosine term
ca	denotes carrier or carrier phase
cf	denotes carrier frequency
co	denotes code
D	denotes down component
D	denotes Doppler measurement
D	denotes database-indicated
d	denotes at the detector
d	denotes dynamic
DC	denotes differentially-corrected
E	denotes early correlation channel
E	denotes Earth's geomagnetic field
e	denotes Earth-centered Earth-fixed co-ordinate frame
F	denotes feature-matching measurement
f	denotes forwards filter
f	denotes front-wheel co-ordinate frame
f	denotes fused solution
f	denotes feature-matching sensor body co-ordinate frame
G	denotes resultant position and time
G	denoted GNSS-derived
G	denotes Gaussian distribution
g	denotes gyro
GNSS	denotes GNSS partition
H	denotes horizontal
h	denotes height
h	denotes hard-iron
I	denotes in-phase
I	denotes ECI frame synchronized with ECEF at time of signal arrival
I	denotes INS-derived
i	generic index
i	denotes Earth-centered inertial co-ordinate frame
i	filter bank hypothesis index
i	denotes applicable to the inclination angle
ic	denotes ionosphere-corrected
IF	denotes intermediate frequency
INS	denotes INS partition
j	generic index
j	satellite or tracking channel number
k	iteration index for Kalman filter or tracking loop
k	generic index

L	denotes late correlation channel
L	denotes latitude
L	denotes left (wheel)
L	denotes leveling measurement
M	denotes magnetic heading measurement or error states
m	denotes Markov process
m	denotes pertaining to a multipath component with respect to the direct signal
m	denotes magnetometer-measured flux density and frame thereof
m	denotes magnetometer
N	denotes narrow-band
N	denotes noise channel
n	denotes local navigation co-ordinate frame
Nav	denotes navigation solution
ND	denotes normalized code discriminator
NED	denotes nominal emission delay
NF	denotes normalized carrier-frequency discriminator
NΦ	denotes normalized carrier-phase discriminator
O	denotes odometer measurement
o	denotes orbital co-ordinate frame
o	denotes odometer
P	denotes position
P	denotes prompt correlation channel
P	denotes PDR measurement
p	denotes precession
p	denotes from the phase modulator
PDR	denotes PDR measurement
Q	denotes quadraphase
Q	denotes quasi-stationary alignment measurement
R	denotes right (wheel)
R	denotes terrestrial radio navigation and measurement thereof
R	denotes reference-navigation-system-indicated
r	denotes applicable to the orbit radius
r	denotes pseudo-range rate
r	denotes rear-wheel co-ordinate frame
r	denotes receiver
r	denotes random walk process
r	denotes reference body co-ordinate frame
ra	denotes accelerometer random noise
Ref	denotes reference navigation system
rg	denotes gyro random noise
S	denotes a point on the Earth's ellipsoidal surface
s	denotes a point on the Earth's geoid surface or water surface
s	denotes static
s	denotes due to or of sculling motion
s	denotes of the Schuler oscillation
s	denotes sub-carrier
s	denotes sine term
s	denotes satellite body co-ordinate frame

s	denotes soft-iron
s	denotes scattering-surface co-ordinate frame
Sensor	denotes sensor
T	denotes the transpose of a matrix
T	denotes time
T	denotes TRN measurement
t	denotes due to tracking noise
t	denotes transmitter or transmitter body frame
t	denotes terrain
TD	denotes time difference
u	denotes applicable to the argument of latitude
V	denotes velocity-matching transfer alignment measurement
v	denotes oscillatory/ vibratory
v	denotes velocity
VE	denotes very early correlation channel
VL	denotes very late correlation channel
W	denotes wide-band
w	denotes wander-azimuth co-ordinate frame
w_lag	denotes lag induced tracking error
x	denotes first component of a vector or axis
x	denotes cross-track component of velocity
y	denotes second component of a vector or axis
Z	denotes ZVU measurement
z	denotes third component of a vector or axis
α	denotes a generic object frame
α	generic index
β	denotes a generic reference frame
β	generic index
γ	denotes a generic set of resolving axes or frame
γ	generic index
δ	denotes a generic co-ordinate frame
ΔR	denotes radio navigation delta-range measurement
$\Delta\Delta$	denotes double-delta discriminator
Δo	denotes differential odometer
δx	denotes state vector residual
δz	denotes measurement innovation/residual
λ	denotes longitude
ρ	denotes range or pseudo-range
ρrc	denotes receiver clock offset
Σ	denotes a summation
ψ	denotes attitude/ GNSS attitude measurement
0	denotes value at the geoid
0	denotes initialization value
0	denotes a constant value
0	denotes at the reference time
0	denotes samples after carrier correlation and before code correlation
–	denotes after state propagation and before measurement update

+	denotes after measurement update
$\perp$	denotes perpendicular

See also the list of acronyms and abbreviations

Qualifiers

E()	expectation operator
(r)	denotes reference-station-indicated
(u)	denotes user-indicated
δ	denotes a small increment or error
Δ	denotes an increment, error or change
$'$	denotes alternative version
$''$	denotes alternative version
$\hat{}$	denotes estimate
~	denotes a navigation system measurement
$\bar{}$	denotes average value
(–), –	denotes at the beginning of the navigation processing cycle
(+), +	denotes at the end of the navigation processing cycle