PRINCIPLES OF
GNSS, INERTIAL, AND MULTISENSOR INTEGRATED
NAVIGATION SYSTEMS

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On-line Appendix C

Historical Navigation Systems

6 April 2008	VLF.
19 April 2008	Decca.
20 April 2008	Loran.
5 May 2008	Introduction, From Navigation Aids to Navigation Systems, Transit and Tsikada, Other Radio Navigation Systems.
10 May 2008	Corrections and extra figures.
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ON–LINE APPENDIX C

Historical Navigation Systems

There are three reasons for adding a historical appendix to a book on contemporary navigation systems [0]. The first is simply completeness, the second is to put contemporary systems in context, helping to draw parallels between the different technologies, while the final reason is that some of the techniques used in past systems might be applicable in future systems. The focus of this appendix is limited to electronic navigation systems. For older navigation techniques, the reader is directed to [1].

As well as obsolete systems, this appendix also includes navigation systems which are still operational, but have a limited user base and may be discontinued within the next few years; they are not covered in [0].

Section C.1 discusses the general evolution of navigation technology over the decades since World War II from aids to a human navigator to systems providing a complete navigation solution. The remaining sections then describe a selection of obsolete and obsolescent navigation technologies. Section C.2 covers very low frequency (VLF) navigation systems, including Omega. Section C.3 describes the Decca Navigator System and its offshoots, while Section C.4 covers the legacy versions of Loran. Section C.5 describes the original satellite navigation systems, Transit and Tsikada. Section C.6 then summarizes the main features of a number of other radio navigation systems, both terrestrial and satellite. Lastly, Section C.7 discusses early inertial navigation technology.

C.1 From navigation aids to navigation systems

Like any technology, the performance of electronic navigation systems has improved over the decades, while cost, size, mass and power consumption have dropped. Improvements have been both evolutionary, retaining backwards compatibility, and revolutionary, requiring new equipment to replace old. However, a key trend has been once of increasing automation.

Navigation of a ship or aircraft was originally performed by a human, known as a navigator. He determined the host vehicle's position from landmarks, by observing the stars and using dead-reckoning sensors, such as a ship's log or airspeed indicator, combined with a magnetic compass [1].

Radio signals were first used simply as electronic landmarks. The navigator determined the direction of the transmitter by rotating the receiver's antenna (either physically or electronically) and identified it by its frequency and/or a Morse call sign or program content. These transmitters were known as *navigation aids* or *aids to navigation* as they helped the human navigator to determine position.

During World War II, a host of new navigation technologies were developed by a range of countries. As they needed to be brought into operation quickly, they were crude and often tailored to particular applications. Use of these systems required listening out for Morse codes or observing traces on oscilloscopes.

Following the war and through the 1950s, these technologies were improved and standardized across countries. A suite of systems: Decca, DME, Lorans A and C and VOR, some of which are still in operation, took over. These provided navigators with range, bearing or range difference measurements directly. For the hyperbolic systems, users were supplied with maps on which LOPs were printed, enabling them to determine position relatively easily from the measurements output by the user equipment.

The development of transistor-based digital computing through the 1960s enabled the computation of the position solution to be automated. Decca even produced equipment that displayed position on a map. At about the same time, inertial navigation, Omega and Transit were introduced, providing global electronic navigation for the first time and finally rendering astronomical navigation largely obsolete. The navigators role was reduced to initializing the systems and selecting transmitters. Arguably, this marked the point at which navigation aids became navigation systems.

Through the 1970s and 1980s, the technology remained largely stable, with only evolutionary improvements. However, each navigation technology produced its own position solution and it was left to the human navigator to combine the information and decide which systems were reliable.

By the 1990s (much earlier for guided weapons), the final stage of processing, integration and integrity monitoring of the different navigation sensors had become automated. At the same time, GPS rendered older navigation systems, such as Decca, Omega and Transit obsolete, while DME, VOR and Loran-C were demoted to back-ups. By the end of the 20th century, the role of aircraft navigator had become obsolete. Figure C.1 summarizes the processing stages of an integrated navigation system.

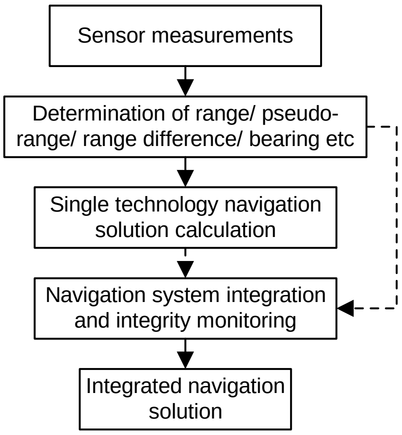


Figure C.1 Integrated navigation system processing stages

In the first decade of the 21st century, the advent of cheap fully-automated navigation systems has massively widened the user base to include land vehicle and personal navigation. However, as GNSS performance in and around buildings is poor, new terrestrial radio navigation technologies are being developed to fill the gap, heralding the era of ubiquitous navigation and positioning.

C.2 Very Low Frequency Navigation

The very low frequency region of the radio spectrum ranges from 3 to 30 kHz in frequency and 10 to 100 km in wavelength. The Earth’s surface and the lowest region of the ionosphere, known as the D layer, act as the boundaries of a waveguide. This enables VLF signals to propagate all of the way around the Earth. Thus, a VLF navigation or communication system can achieve global coverage with very few transmitters. The skin depth of sea water at 10 kHz is about 3 m. Therefore, VLF signals may be received by submarines without the need to surface.

Propagation of VLF signals is complex [2–4]. As with Loran signals (see Section 9.2 of [0]), the propagation speed over land varies with location and seasonally. Variation in the effective height of the ionosphere’s D layer from about 60 km during the day to 90 km at night also introduces a time-varying phase shift, known as the diurnal effect. Further distortions in signal propagation arise due to the Earth’s geomagnetic field and during solar storms. Close to a transmitter, interference from higher-order waveguide modes can occur, while at its antipode, signals traveling via different paths around the Earth will generally interfere with each other. Thus, to get the best position accuracy from VLF signals, those signals should be selected carefully and the various propagation effects accounted for.

Section C.2.1 describes the US-led Omega system, Section C.2.2 describes the Russian RSDN-20/ Alpha system, Section C.2.3 discusses the use of VLF communications signals and Section C.2.4 comprises a brief note on Decca Long-Range Area Cover (DELRAC).

C.2.1 Omega

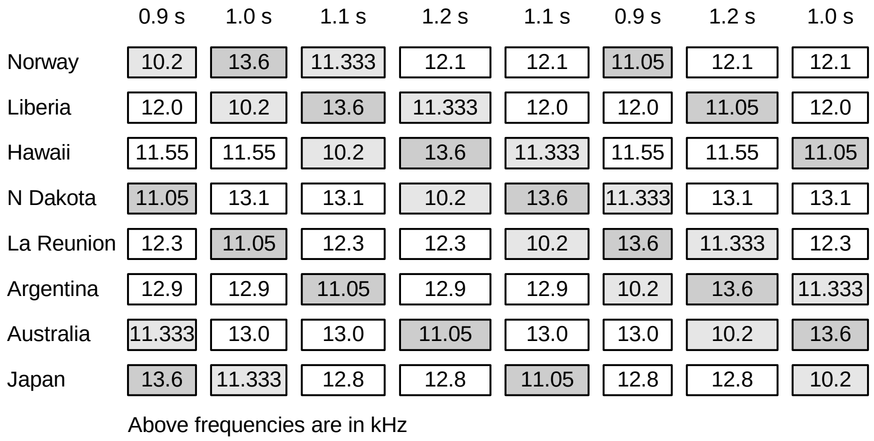
Omega was the first navigation system to provide continuous world-wide coverage. Its origins lay in the DELRAC (see Section C.2.4) and Radux (see Section C.6.9) systems proposed in the 1950s. A nascent Omega system, serving North America and the North Atlantic, was implemented in the 1960s. This used four transmitters in New York state, Trinidad, Hawaii and Norway [3]. Global coverage using eight stations was achieved in the early 1970s with additional transmitters in Liberia, La Reunion (Indian Ocean), Argentina and Japan, while the mainland United States transmitter was moved from New York to North Dakota. In 1982, a new transmitter in Australia replaced Trinidad [2, 4, 5].

Using propagation models and databases, Omega was typically accurate to 2–4 km (1σ), though errors could exceed 10 km during ionospheric storms. It was used by aircraft in mid ocean, where more accurate positioning systems were out of range, and by submarines. Some ships also used it, though many favored Transit integrated with INS. Omega was closed down on 30 September 1997 after most of its user base had switched to GPS.

The original Omega signal format comprised unmodulated continuous-wave signals on three frequencies, 10.2, 11.333 and 13.6 kHz, which were transmitted by each of the eight stations in turn. The transmission cycle was divided into eight slots, or segments, of between 0.9 and 1.2 s, separated by 0.2 s intervals, with each station transmitting on one frequency at a time. The transmission slots were of different lengths to enable user equipment to distinguish between stations. The intervals were provided to allow time for the transmission on that frequency to disperse and for the transmitter antenna circuits to be retuned to the next frequency. All transmitters were synchronized to UTC with an error standard deviation of 2 μs (equivalent to 600 m in range).

With only carrier-phase positioning available, the range ambiguity was 29.4 km at 10.2 kHz, 26.5 km at 11.333 kHz and 22.1 km at 13.6 kHz. For basic user equipment using only the 10.2 kHz signals, the position solution ambiguity could be as low as 14.6 km, while for 2 or 3 frequency operation, the minimum position solution ambiguity was 131.4 km. The ambiguity was resolved by initializing at a known position.

The signal format was upgraded in the late 1970s with a fourth common frequency of 11.05 kHz added. This improved the ambiguity by a factor of 4, with a range ambiguity of 1059 km and minimum position ambiguity of 529 km. Furthermore, a set of transmitter-unique frequencies was added for use during the remaining four transmission slots of each station. With each station transmitting in all of the slots, the signal to noise performance and dynamics response of the user equipment was improved. Figure C.2 shows the transmission cycle.



	0.9 s	1.0 s	1.1 s	1.2 s	1.1 s	0.9 s	1.2 s	1.0 s
Norway	10.2	13.6	11.333	12.1	12.1	11.05	12.1	12.1
Liberia	12.0	10.2	13.6	11.333	12.0	12.0	11.05	12.0
Hawaii	11.55	11.55	10.2	13.6	11.333	11.55	11.55	11.05
N Dakota	11.05	13.1	13.1	10.2	13.6	11.333	13.1	13.1
La Reunion	12.3	11.05	12.3	12.3	10.2	13.6	11.333	12.3
Argentina	12.9	12.9	11.05	12.9	12.9	10.2	13.6	11.333
Australia	11.333	13.0	13.0	11.05	13.0	13.0	10.2	13.6
Japan	13.6	11.333	12.8	12.8	11.05	12.8	12.8	10.2

Above frequencies are in kHz

Figure C.2 Final Omega transmission cycle

Early designs of user equipment tended to measure relative phase differences between signals from the different transmitters (using an internally-generated signal as a "flywheel") and used hyperbolic positioning to determine a navigation solution. Later user equipment constructed a pseudo-range measurement for each transmitter from the phase measurements on the various frequencies and then used passive ranging methods to determine a user position and receiver clock solution [4].

The accuracy of Omega could be improved substantially using differential techniques. By the 1990s, 30 differential Omega stations were in operation around Europe, NW Africa, Eastern Canada, the Caribbean, India and Indonesia. Corrections were transmitted using marine radio beacons, operating between 285 and 415 kHz. Beacon ranges were between 400 km and 1000 km and the differential Omega position error varied from about 300 m (1σ) 200 km from the beacon to 1 km at maximum range [2, 4].

C.2.2 RSDN-20/ Alpha

The Russian version of Omega was known as RSDN-20 or Alpha [6]. RSDN is the Russian acronym for Normal Long-range Navigation System. The original system, developed in the 1960s, used three transmitters at Krasnodar, Novosibirsk and Komsomolsk, all in Russia; these were collectively known as Sigma. Interestingly, the signal geometry was too poor to obtain a navigation solution within most of Russia, but was excellent over the Arctic, North America, China and Japan.

Three common frequencies were used, 11.905, 12.649 and 14.881 kHz, with the system occasionally using 12.500, 13.281 and 15.625 kHz instead. The range ambiguity using the 3 main frequencies was 403 km. The transmission cycle was 3.6 s, divided into six 0.4 s transmission slots, interspersed by 0.2 s intervals, with each transmitter using different frequencies in different slots. Each frequency and transmitter was silent for a different part of the transmission cycle to allow the stations to be distinguished.

RSDN-20 was upgraded in 1991 with two new transmitters in Revda and Seyda, while the Komsomolsk transmitter was moved to Khaborovsk. This extended coverage to Europe and Russia itself. Revda and Seyda broadcast slots at 12.091 and 12.044 kHz, respectively, as well as using the main frequencies. RSDN-20 was still operational as recently as 2006.

C.2.3 VLF Communications

Many Omega receivers also made use of VLF communications stations operating between 15 and 25 kHz [2]. VLF communications stations do not frequency hop, though the signal modulation can be used to resolve the range ambiguity inherent in carrier phase measurements. However, these stations were not synchronized to UTC, so they were used only for range-rate of delta range measurements. Most VLF communications stations have been closed since Omega was decommissioned in 1997 as aircraft and ships have switched to satellite systems. However, a few remain operational for submarine use at depths down to about 15 m. For deep submarines, the US and Russia operate communication systems at the much lower frequencies of 76 and 82 Hz, respectively.

C.2.4 DELRAC

DELRAC was a long-range VLF navigation system proposed by the Decca Navigator Company in 1954 [7, 8]. DELRAC was never built, being dropped in favor of DECTRA (Section C.3.3). However, its signal format formed the basis for Omega and RSDN-20 and the Decca company successfully sued the US government for patent infringement in the 1970s [9].

C.3 Decca Navigator System

The Decca Navigator System, usually referred to simply as Decca, was a medium to long-range low-frequency (LF) radio navigation system, which operated between 1946 and 2001. Transmitters were grouped into chains of four, comprising the master, red slave, green slave and purple slave. After 1973, a few chains operated with only three transmitters, usually omitting purple. Each slave transmitter was phase-synchronized to the relevant master, but there was no synchronization between chains. All measurements made by the receiver compared a slave with its master. Hyperbolic positioning was then used to determine a latitude and longitude solution. Most Decca user equipment could only receive one chain at a time. However, multi-chain receivers were available. Coverage of each chain was up to about 400 km from the master station [2, 7, 10–12].

Altogether, 55 Decca chains were operational over the history of the system. Figure C.3 shows the number of chains operating each year [13]. The first chain, opened in 1946, was the English chain, reflecting the development of Decca within the UK. This was followed by chains in Denmark, Germany, France and other parts of the UK. In 1957, a series of chains were opened in Eastern Canada, with a New York chain opening the following year. Decca coverage peaked in 1980, by which time all of North and West Europe, Japan and South Africa had been added, together with California, Bangladesh, the United Arab Emirates, Nigeria and parts of India and Australia.

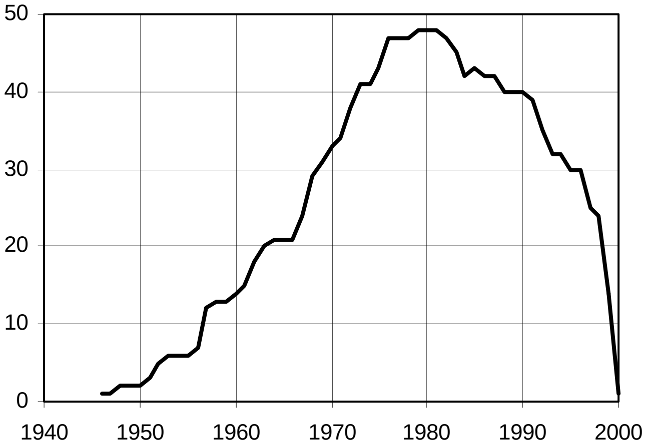


Figure C.3 Decca coverage, in chains, over the history of the system

The decline of Decca began in the early 1980s when the North American chains were switched off, following the adoption of Loran-C. Further countries followed, but in others, such as Decca's native UK, the system remained popular until the completion of the GPS constellation in the 1990s. A co-ordinated shutdown of the remaining chains in Europe and South Africa occurred at the end of 1999 and spring 2000. The final chain, in Japan, closed in March 2001.

Decca was used primarily by the commercial shipping and yachting communities in coastal areas, but was also used in helicopters and light aircraft. Section C.3.1 describes the Decca signals, while Section C.3.2 discusses the system accuracy. Section C.3.3 describes Decca Track (DECTRA).

C.3.1 Signals

Decca used five LF frequency bands: 70.087–70.583, 84.105–85.900, 112.140–114.533, 114.943–117.397 and 126.157–128.850 kHz. Unlike Loran, Decca was a FDMA system. Each chain used one frequency within each band, based on a fundamental frequency, f, such that the chain's transmission frequencies were $5f$, $6f$, $8f$, $8.2f$, and $9f$. 63 different sets of frequencies, known as codes, were used.

For most of the Decca duty cycle, each transmitter broadcast an unmodulated carrier on one frequency, with the master and the red, green, and purple slaves using the $6f$, $8f$, $9f$, and $5f$ frequencies, respectively. LOPs were determined by comparing the slave carrier phase with that of the master, noting that the measurement was referred to by the slave color. However, with each transmitter on a different frequency, the ambiguity was at the wavelength corresponding to the lowest common multiple of each pair of frequencies. Consequently, the minimum separation of candidate LOPs, known as the lanewidth, was 351 m for purple measurements, 439 m for red measurements and 585 m for green measurements. The original Decca design relied on a known starting position for ambiguity resolution, then known as lane identification. However, the system was vulnerable to cycle slips, particularly for air users. Therefore, the signal was modified.

The first modified signal was known as V mode and involved the interruption of the regular Decca signals for three ~half-second intervals each minute, during which the master transmitted on the 5*f* and 6*f* frequencies and one of the slaves on the 8*f* and 9*f* frequencies. This enables the minimum LOP ambiguity to be increased to 10.525 km. The separation of these LOPs was known as a zone.

V mode was superseded by multipulse, whereby the main signals were interrupted for four 0.75 s intervals every 20 s to enable each station in turn to transmit on all five frequencies for 0.45 s. This multipulse increased the minimum LOP ambiguity to 52.625 km and was also known as zone identification. During the remainder of the duty cycle, transmissions on 8.2*f*, known as the orange frequency, were distributed amongst the four transmitters, with some time slots used for data but transmission.

As Decca's main transmission mode occupied a much larger portion of the duty cycle, the resolution was much finer than for the multipulse or V mode transmission. Thus, the two types of Decca signal were often known as fine and coarse.

C.3.2 Accuracy

Decca signals were subject to the same propagation errors as Loran-C signals as described in Section 9.2.4 of [0]. However, the continuous wave nature of the Decca signals, compared to the pulsed nature of Loran signals, brought two consequences. The resolution of Decca was better at around 5 m. However, the sky wave component of the signal could not be separated from the ground wave in the receiver. Consequently, Decca performance at long ranges and at night was much poorer than that of Loran-C.

The specified Decca position error thus varied from less than 200 m during the day within about 100 km of the master station to more than 2.5 km at maximum range during a winter night [2].

C.3.3 DECTRA

DECTRA was a long-range enhancement of Decca, making use of the master and purple slave transmissions on the 5*f* frequency from two distant Decca chains [7, 14]. One chain was phase-synchronized to the other. Accuracy was relatively poor at around 6 km along the track between the two chains and 15 km across the track. Only one DECTRA chain was ever implemented, linking the Newfoundland Decca chain in Canada with the Scottish chain in the UK. It was commissioned in 1957 and operated for about 10 years before being rendered obsolete by civil aviation's adoption of inertial navigation.

C.4 Loran

This section describes the main features of the Loran systems that preceded the current ELoran standard. Lorans A, B, C and D are described in turn, along with Cytac and Pulse 8, followed by skywave-synchronized (SS) Loran.

C.4.1 Loran-A

The first version of Loran was developed by the US during World War II, with the first full-scale trial in 1942. It was originally known simply as Loran, before becoming standard Loran and then Loran-A as subsequent versions became established. By 1945, Loran-A transmitters at 72 sites were in operation, with only a few added subsequently. Most of the coastal areas of the North Atlantic, North Pacific and Caribbean, extending about 1000 km out to sea, were covered. Land coverage was much more limited. Closure of the Loran-A stations started in the 1970s as Loran-C took over. The last Atlantic transmitters were switched off in 1980, with a few Pacific transmitters surviving until the early 1990s [9, 11].

Loran-A transmitters were grouped in synchronized pairs, rather than chains, though most sites comprised transmitters for two pairs. Transmissions comprised a single 45 μs pulse, with the master station pulse triggering transmission of the slave station pulse [7, 9, 11, 15]. Hyperbolic LOPs were determined by measuring the difference in time of arrival of these two pulses and subtracting the difference in time of transmission. The pulses were short enough to enable the ground and sky wave components of the signal to be distinguished as in later versions of Loran. Most signals were transmitted on 1950 or 1850 kHz, though 1900 and 1750 kHz were also used. The pulse repetition interval varied between 29.3 and 50 ms. The position accuracy of Loran-A was typically 2–3 km, dominated in later years by the accuracy of the charts then used to determine position from the time difference measurements. The timing resolution was about 1 μs, corresponding tp 300 m of range, for both the transmitter to receiver and master to slave links.

C.4.2 Loran-B

Loran-B operated between 1948 and 1955 as an experimental system only. Based on Loran-A and using the same frequencies, it phase synchronized the pulses to the carrier to enable a much higher timing precision. Loran-B also enabled the transmitters to operate in chains rather than pairs. It was superseded by Loran-C [9, 13].

C.4.3 Loran-C and Cytac

Loran-C was developed in the late 1950s, evolving out of the Cytac system, which replaced Cyclan (Section C.6.2). Cytac operated on 100 kHz with a mixture of phase comparison and pulsing. It was tested in 1955 [9, 11].

The first Loran-C chains opened in 1957. The single pulses of Lorans A and B were replaced with groups of eight or nine, improving the signal to noise. However, the largest change was the switch to the low frequency of 100 kHz, increasing the coverage radius by about 50% over sea and more than a factor of ten over land.

It is a matter of opinion as to whether Loran-C should count as a historical system. Because its replacement, ELoran, is backwards compatible, legacy Loran-C user equipment will continue to operate indefinitely. See Chapter 9 of [0] for more information.

C.4.4 Loran-D and Pulse-8

Loran-D and Pulse-8 were shorter-range higher-accuracy versions of Loran-C, operating on the same frequency and using much more closely spaced transmitters. Loran-D, a military system, used transportable transmitters, and broadcast 16 pulses per group, instead of 8. The extra pulses were interleaved between the regular pulses and modulated so that a standard Loran-C receiver would ignore them. It was used for NATO military exercises, including those within Europe, in the 1960s and 1970s [9, 13].

Pulse-8 was a privately operated system, run by the Decca Navigator Company. It was used in the 1970s and 1980s for positioning in the North Sea and other offshore areas. It shared the same signal format as Loran-C, but was able to achieve an accuracy of 25 m (1σ), largely by using only signals propagated over water so that the propagation speed was well known [13].

C.4.5 Skywave-synchronized Loran

Skywave-synchronized (SS) Loran was a version of Loran-A with pairs of transmitters separated by a few thousand kilometers and the slaves synchronized to the masters using skywave signals instead of the normal ground wave. The system worked only at night, but provided extensive coverage over land as well as sea. Accuracy was similar to standard Loran-A. SS Loran was used towards the end of World War II for bombing over Europe [9].

C.5 Transit and Tsikada

The US Transit and Russian Tsikada (meaning cricket) were the first satellite navigation systems and the first radio navigation technologies with global coverage, albeit not continuous. The two systems were very similar in design. They were used for marine navigation, including by submarines (with an antenna at the sea surface), and for surveying [2, 16].

Both systems used satellites in low polar orbits, such that only one satellite was generally visible at a time. All satellites broadcast on the same frequencies, approximately 150 and 400 MHz, and signals comprised carrier and navigation data, but no ranging codes. Thus, where more than one satellite was visible at a time, they would interfere with each other. For marine navigation, inertial navigation or another dead-reckoning technique was used to determine position between Transit/Tsikada fixes.

Positioning in the horizontal place was accurate to about 25 m on a single pass where the velocity and height were well known and a dual frequency receiver was used. Where these conditions were not met, the system still worked, but performance was degraded. For surveying using multiple passes and differential techniques, an accuracy of around 5 m could be achieved.

Sections C.5.1 and C.5.2 describe the features unique to Transit and Tsikada, respectively, while Section C.5.3 describes the Doppler positioning technique used by both systems.

C.5.1 Transit

Transit was also known as the Navy Navigation Satellite System. Development started in 1958 as a US Navy system for submarine use. The first experimental satellite was launched in 1961 and the system became operational at the beginning of 1964. It was opened to civil use from 1967 and decommissioned in 1996 after being superseded by GPS. Between four and seven satellites were operational at a given time. Orbits were circular with a radius of 7440 km, 1075 km above the Earth’s surface, and a period of 107 minutes. Transit satellites were visible for 10–18 minutes per pass [2]. Navigation data was modulated at ~50 symbol s^{-1} by phase shifting the carrier. Only some of this data was available to civil users.

C.5.2 Tsikada

The first Tsikada satellite was launched in 1967. Separate military and civil systems, using different satellites, were maintained. The military system, known as Tsikada-M or Parus, became fully operational in 1974 and continues in use today, with the most recent satellite launch in September 2007. The Parus system is also used for communications. The constellation originally comprised eight civil satellites, but now comprises four. The four-satellite civil system, known simply as Tsikada, opened in 1978 and closed in November 2003.

The orbital radius was 7165 km, 800 km above the Earth’s surface, and the period was 105 minutes. Navigation data was modulated at 50 symbol s^{-1} by varying the frequency of an amplitude-modulated tone.

C.5.3 Doppler Positioning

Doppler positioning for Transit and Tsikada used signals from a single satellite. This required measurements to be taken over a period of time, long enough for the signal geometry to change sufficiently for the different components of position to be separated. The Doppler frequency shift of the signal was measured and then integrated to give a delta range measurement. Assuming the satellite position and velocity was known from the navigation data message and the user velocity was known from and INS or dead-reckoning system, four delta range measurements across a satellite pass were required to solve for a full position solution (at a given point in time during the pass) and the user–satellite relative clock drift. Figure C.4 illustrates this.

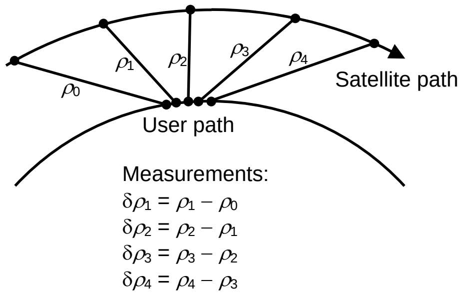


Figure C.4 Single-satellite Doppler positioning geometry

Transit and Tsikada satellite orbits were polar with the ground tracks inclined at about 4° to a meridian due to the Earth's rotation. Consequently the dilution of precision for determining latitude over the course of a satellite pass was much better than that for longitude and height. However, the height was often known for marine navigation. Therefore, by constraining this within the navigation solution, the accuracy of the longitude solution could be improved. If the satellite passed directly overhead, longitude could not be determined at all, so these satellites were not generally used.

Best signal to noise performance was obtained by tracking the signals continuously during a satellite pass. The measurements could be processed as four delta range measurements, each taken over several minutes, or as multiple delta range measurements, taken over shorter intervals. The latter approach would make position determination more complex, but enable variations in the relative clock drift over the course of a pass to be modeled.

Errors in the user velocity solution used to aid position computation could significantly degrade position accuracy. For Transit, a velocity error of 1 m s^{-1} lead to a position error of around 700 m [16]. Therefore, effective Doppler positioning with Transit or Tsikada on a moving vehicle required integration with a high-grade INS or other dead-reckoning system, in contrast to the relatively poor sensors often integrated with GNSS today.

The Argos satellite system, first developed in 1978 by Centre National d'Etudes Spatial (CNES) of France and the National Oceanic and Atmospheric Administration (NOAA) of the USA, also used Doppler positioning. However, as this was primarily a tracking system, rather than a navigation system, the mobile users transmitted and the satellites received.

C.6 Other Radio Navigation Systems

This section briefly summarizes a range of other radio navigation systems, including experimental systems. Consol, Cyclan, direction finding, Gee, Geostar and Locstar, the Microwave Landing System (MLS), Navaglobe and Navarho, Radio Mailles, Radio-range and Two-course Beacons, Radux, Rebecca-Eureka and Shoran, and Starfix are all described. Systems are listed in alphabetical order.

Other historical systems, not described here, include Condar, Electra, Navar, Post Office Position Indicator (POPI) and Visual Aural Range (VAR). There were also proposals to combine navigation and radar technology under programs such as Black Maria, Lanac and Teleran [17].

C.6.1 Consol

Consol was a bearing-indicating beacon system. The range was about 1500 km. Each station transmitted two directional signals with antenna patterns which varied with time over a 40–60 s duty cycle, beginning with an omnidirectional synchronization signal. Both directional signals were on the same frequency and were distinguished by a Morse dash modulation on one signal and a dot on the other. By counting the number of dots or dashes between the synchronization signal and the point at which the two signals ere equal, the bearing could be determined to an accuracy of 0.6° with an ambiguity of 7.5° [7, 11, 18].

Consol was developed by Germany as Sonne (meaning sun) and was commissioned in 1940 with two transmitters in Spain and one in Norway. The system was retained after World War II and renamed Consol. Transmitters in France and Northern Ireland were added. Between 1958 and 1960, the USA and USSR opened two transmitters each. The American version was known as Consolan. Consol broadcast between 257 and 363 kHz, while Consolan used 192 and 194 kHz. These signals could be received on a consumer AM radio equipped with a long-wave band, so the system continued to be used by pleasure craft, fishing vessels and light aircraft after ships and larger aircraft had moved onto more sophisticated systems. Most Consol stations were switched off by 1980 [9].

C.6.2 Cyclan

Cyclan was an experimental phase-comparison hyperbolic navigation system, demonstrated in 1946 by the Sperry company in the USA. It used two frequencies in the LF band, 180 and 200 kHz. Stations transmitted in turn with two frequencies used to aid ambiguity resolution. This, in turn, was replaced by Cytac, which evolved into Loran-C (Appendix C.4.3) [9, 11].

C.6.3 Direction finding

Direction finding is the oldest form of radio navigation. It can operate using any transmitter, provided the location is known and it can be identified. Early direction finders rotated a loop receiving antenna to determine the direction in which the received signal strength was a minimum. Later designs used a pair of orthogonally mounted loop antennas. The outputs were summed with the weighting and relative sign varied to minimize the received signal strength. The direction of the signal was then obtained from the received signal strength [4, 19].

Both aircraft and ships used LF and MF transmissions for direction finding. For ships these provided the longest range, while for aircraft, it was not possible to design effective VHF and UHF direction finding equipment due to a combination of airframe reflection and refraction and the need for an aerodynamic antenna design. The range was limited to a few hundred kilometers at LF (less at MF) by the need to avoid skywave interference. Aircraft used a mixture of broadcast stations and dedicated non-direction beacons (NDBs). Ships have tended to use dedicated marine radio beacons as best range is obtained by locating these on the coast, while most high-power broadcast transmitters are located inland. In recent years, most marine and many inland radio beacons have been used as differential GNSS data-links.

LF and MF direction finding equipment is typically accurate to about 2°. However, errors caused by the host vehicle’s structure can degrade this to about 5°. There is a 180° ambiguity in the direction of a single transmitter. However, this may easily be resolved by measuring the directions of two transmitters as the line of sight vectors only intersected at one place. A two-dimensional position fix may be obtained by direction finding from a single transmitter if measurements are taken at different times and the host vehicle’s relative motion between measurements can be obtained from a dead-reckoning system. If signals from three transmitters are available, direction finding may be used to determine the vehicle heading as well as its horizontal position; however, the mathematics is complicated and other methods are usually more accurate.

Dedicated NDBs are scheduled for withdrawal from 2010 onwards, while many AM broadcast radio transmitters have already been closed. SBAS is likely to supersede the use of beacons for differential GNSS, though their use for augmenting ELoran has been proposed [20].

C.6.4 Gee

Gee was developed in the UK during World War II and continued in operation until 1970. Transmitters were grouped into chains of 3 or 4 transmitters, like Loran-C and Decca, and positioning was hyperbolic. There were four chains in the UK, two in France and one in Germany [9].

Gee used VHF frequencies between 22 and 85 MHz, so coverage was line of sight. Its range varies from about 300 km at sea level to 650 km at high altitude. It was used mainly by aircraft. Signals comprised 0.6 s pulses, repeated on a ~ 500 Hz cycle. Station A, the master, transmitter every cycle, stations B and C transmitted on alternate cycles, while station D broadcast double pulses. Station A broadcast additional pulses every 4th cycle to enable the 'B' and 'C' cycles to be distinguished. The slave stations transmitted their pulses at a fixed interval after receipt of pulses from the master station. Gee was accurate to a few hundred meters [7].

Gee was cast aside in favor of VOR/DME and TACAN because, prior to the advent of digital computers it was easier to navigate in terms of range and bearing to a series of waypoints that to determine a hyperbolic position solution.

C.6.5 Geostar and Locstar

Geostar and Locstar were proposed radio determination satellite service (RDSS) systems, operating on the same two-way ranging principle and using the same frequencies as the first-generation Beidou system described in Section 6.5.1 of [0]. Geostar was to cover North America, while Locstar was to cover Europe, the Middle East and Africa [2]. However, as the systems offered horizontal-only position with a limited update rate and would have charged a monthly subscription, they were superseded by GPS before the satellites were launched.

C.6.6 Microwave Landing System

The Microwave Landing System was developed in the 1970s and 80s as a replacement for ILS (see Section 9.3 of [0]) [2, 5]. Each MLS installation used a single frequency in the 5031.0–5090.7 MHz band, with the azimuth, elevation and data components time multiplexed. The localizer and glideslope of ILS were to be replaced by scanning beams about 1° wide. By timing the interval between receiving the maximum signal strength in the to and fro sweeps of the beam, users determined their bearing and elevation relative to the recommended approach route. Range was to be provided by a modified form of DME, known as DME/P, which offered higher precision from use of a larger bandwidth, but shorter range, than conventional DME.

MLS is currently in a state of limbo. A number of experimental MLS installations were deployed at US airports in the 1980s. However, they were switched off around 2000 when the decision was made to develop GNSS-based landing systems instead. Significant improvements in ILS performance were also achieved while MLS was under development. However, the US Air Force has continued to use MLS, while a variant known as the Microwave Scanning Beam Landing System (MSBLS) was used for Space Shuttle landing. In 2003, an MLS installation was deployed at Heathrow airport in London, together with user equipment on some British Airways aircraft [21]. Wider deployment in the UK and France is currently under consideration.

C.6.7 Navaglobe and Navarho

Navaglobe was a bearing-only system operating in the 70–150 kHz band. Development started in 1946 in the USA with a demonstration in 1954. Each beacon switched between four transmission patterns over a 1 s duty cycle: an omnidirectional pattern and three bi-directional patterns, offset from each other by 60° in bearing. Signals were continuous wave. Users determined their bearings from the beacon by comparing the received signal strengths of the four transmission patterns, noting that there was a 180° ambiguity. The range was about 4000 km and the accuracy was 4° from individual measurements and 1° with time averaging [22].

Navarho superseded Navaglobe, combining it with a separately-developed range measurement technique known as Facon. It was first tested in 1956. Range determination was by passive ranging using carrier phase measurements. Ambiguity resolution and synchronization of the receiver clock to the transmitter was performed by initializing the user equipment at a known location. However, to maintain the proposed ranging accuracy of around 10 km over the course of an aircraft flight of up to 12 hours, a receiver clock oscillator accurate to 1 part in 10^9 was required [7, 22]. this is more accurate that the vast majority of today's GNSS receiver oscillators and would have required Navarho receivers to incorporate atomic clocks. Consequently, Navarho never reached operational status.

Hyperbolic versions of Navarho, known as Navarho-H and Navarho-HH were also proposed [23]. These synchronized the transmitters, avoiding the need for a precision receiver clock. Navarho-H combined a two station hyperbolic LOP with a bearing to obtain user position, while Navarho-GG used three stations to provide two hyperbolic LOPs (like Loran A-D and Decca). Neither system was adopted.

C.6.8 Radio Mailles

Radio Mailles (which translates as Radio Mesh or Radio Web) was an experimental navigation technique developed in France in the 1950s [7, 24]. A demonstration system in the Paris area operated on frequencies around 2 MHz. Four synchronized transmitters, each operating simultaneously on a different frequency, were located in the corners of the coverage area. The signals were modulated at 384 or 385 Hz, with the diagonally opposite transmitters carrying the same modulation. At a given location, the phase difference in the modulation received from neighboring transmitters varied over a 1 s cycle. The time at which the phase difference was zero, known as an isophase, was measured for each of the four pairs of transmitters. These were then subtracted to produce two time differences, from which two near-perpendicular LOPs were obtained.

The baseline between transmitters for the Paris system was 100 km. With a 1 Hz modulation difference, this could have been increased to 390 km without ambiguity. For larger baselines without ambiguity, a smaller modulation difference could be used. However, this would be at the expense of a longer duty cycle and consequently slower update rate, though multiple modulation frequencies might have resolved this. The coverage area could also be extended with additional transmitters. A transmitter could serve multiple coverage areas, so a double coverage area would require 6 transmitters and a 2×2 quadruple coverage area 9 and so forth.

C.6.9 Radio-range and Two-course Beacons

From the 1920s and 1930s, airways in the USA and some other countries were delineated by MF radio beacons, known as Radio-range or four-course beacons. These transmitted directional signal patterns, such that users along four directions, each approximately 3° wide, would receive a continuous tone on an AM radio, whereas other users would receive either a Morse A or N, depending on which sector they were in. Aircraft flying directly towards or away from a beacon could thus maintain course by keeping within the continuous tone regions, with the signal received elsewhere indicating whether they should steer to the left or right. The range over land was 40–500 km, depending on the beacon design [7].

In the UK, two-course beacons were used. These operated on VHF frequencies in the 30–40 and 100–124 MHz bands. Continuous tones were received in only two directions with a Morse E or T received in the remaining sectors [7]. Both systems were superseded by VOR in the 1950s.

C.6.10 Radux

Radux was an experimental long-range hyperbolic navigation system, developed in the USA in the late 1940s [3, 8]. Ranging was performed using a 200 Hz signal modulated onto LF carriers operating around 40 kHz. The carrier frequency differed between transmitters, while the 200 Hz modulation was synchronized. The accuracy was around 4 km (1σ).

Using carrier-phase positioning to improve precision was considered. However, the modulation could not be measured to sufficient accuracy to resolve the ambiguity on the carrier measurements without long averaging periods (the same problem occurs with GNSS). Therefore, around 1950, it was decided to add an additional 10 kHz continuous wave signal to improve accuracy, with each station transmitting in turn for 1 s. The combined system was known as Radux-Omega. As the VLF signals carried further than the 40 kHz signals, the Radux component was dropped later in the 1950s and the Omega component evolved into the system described in Section C.2.1.

C.6.11 Rebecca-Eureka and Shoran

Rebecca-Eureka was a two-way ranging system, developed during World War II by the UK and used in aircraft. Rebecca was the name given to the user equipment, while Eureka was the beacons [7]. It operated in the 190–240 MHz band. Rebecca equipment transmitted about 300 45 μs pulses per second with the beacons responding at the same repetition rate on a different frequency. Eureka beacons provided no bearing information, but Rebecca receivers used twin directional aerials on either side of the aircraft, enabling users to home in on the beacons using direction finding. The range of a beacon was about 400 km at high altitude, while the accuracy was in the 0.5–5 km range, depending on distance. User equipment for the shorter range Blind-approach beacon system (BABS) was often integrated into Rebecca units. The system was replaced by DME in the 1950s, except for Australia, where its use continued into the 1970s.

Short-range navigation (Shoran) was a two-way ranging system for aircraft developed during World War II by the USA. Aircraft user equipment determined position by transmitting pulses to two ground stations and timing the reponses. The system operated in the 200-300 MHz band and was accurate to a few meters. Despite the very high accuracy for its time, it was superseded by VOR/DME and TACAN for aircraft navigation. However, Shoran and its successor, Hiran, continued to be used for surveying [17, 25].

C.6.12 Starfix

Starfix was a satellite navigation system serving the oil industry in the Gulf of Mexico from 1986 until sometime in the 1990s [2]. It used three geostationary satellites, transmitting GNSS-like signals at around 4 GHz. The chipping rate was 2.4576 Mchip s^{-1}, with the code length matched to the data message rate of 150 symbol s^{-1}. Differential correction information was included in the data message. Positioning was horizontal only with an accuracy of about 5 m where the user height was known. Starfix was superseded by the Omnistar DGPS system.

C.7 Inertial Navigation and Attitude Determination

This section discusses 20th century navigation and attitude determination systems based on inertial sensors. Pre-dating today's strapdown technologies, these used mechanical inertial sensors with axes aligned with the local horizontal and vertical. The gyrocompass, directional gyro and vertical reference attitude determination systems are described first, followed by a discussion of early inertial navigation systems and stellar-inertial navigation. Pendulous accelerometers and spinning mass gyros are respectively described in Sections 4.1.1 and 4.2.1 of [0].

C.7.1 Gyrocompass

The gyrocompass was used for heading determination in ships from the early part of the 20th century [7]. It consists of a single large spinning mass gyro with its spin axis aligned along the north-south axis within the horizontal plane. The casing of the gyro assembly is either linked to the casing of the gyrocompass unit via a set of gimbals or floated in a bed of mercury. This isolates it from the motion of the host ship. The heading of the ship can thus be determined by reading off the angle between the gyro spin axis and the fore-aft axis of the ship.

The gyro spin axis stays fixed with respect to inertial space (instrument errors excepted). Therefore as the Earth rotates and the ship moves, it will precess away from alignment with the north-south axis. However, the gyro case is imbalanced with the bottom heavier than the top. As a consequence, the torque exerted by gravity on the gyro case acts as a restoring force to precess the spin axis back into alignment. Because the angular motion of a spinning mass gyro is about the directional mutually orthogonal to the spin axis and applied torque (see Section 4.1.1 of [0]), the direction of the spin axis "spirals in" to alignment with north-south rather than moving directly into alignment.

If the host ship is stationary or moving in an east-west direction, the torque on the gyro as the Earth rotates will cause the spin axis to align itself with north-south regardless of the initial orientation. Thus, a gyrocompass is self-aligning. This removes the need for an initial alignment process and ensures that the gyro sensor errors do not cause the gyrocompass heading error to grow with time.

However, if the ship is traveling in a north-south direction, the "homing" alignment of the spin axis is displaced from north-south by an amount proportional to $v_{eb,N}^{n} / \cos L_b$. At mid latitudes, this equates to about 1° of displacement for a north-south speed of 20 m s^{-1}. As the ship's velocity will be known, this "steaming error" may be compensated either by applying a correction to the gyrocompass output or by applying a restoring torque to re-align the gyro spin axis. However at aircraft speeds, this can not be effectively compensated, so a gyrocompass can not be used. Gyrocompasses also don't work at the poles. However, this has not historically been a problem for shipping.

C.7.2 Directional Gyro

A directional gyro was used to aid aircraft heading determination prior to the advent of the INS and low-cost AHRS [26]. It consists of a spinning mass gyro mounted inside a body axis *xy* plane gimbal with its spin axis in that plane. The gimbal and spin axis are thus in the horizontal plane when the aircraft is level. Conservation of angular momentum keeps the spin axis aligned with respect to inertial spacc. Reading off the direction of the gyros spin axis within the gimbal thus gives the aircraft's heading when it is level. This heading reading is in error when the aircraft pitches or rolls, but recovers when it levels off.

The direction of the gyros's spin axis will drift with time due to instrument errors, while the orientation of the north-south axis in the horizontal plane with respect to inertial space varies as the earth rotates and the aircraft moves. Therefore, a second gimbal, mounted inside the read-out gimbal with its axis mutually perpendicular to the spin axis and the other gimbal, is used to apply torque to rotate the spin axis within the *xy* plane to keep it aligned with north-south (or east-west). The control signal for the torquer may be obtained by comparing the heading reading with a magnetic compass when the aircraft is level. Alternatively, it may be calculated as a function of position and velocity to compensate the rotation of the local navigation frame with respect to inertial space, leaving the instrument errors uncompensated. In this latter case, better quality gyros, with 0.25–0.5°/hr drift were used and alignment was performed on the ground prior to take-off.

C.7.3 Vertical Reference

An averaging vertical reference was used for aircraft pitch and roll determination prior to the advent of the INS and low-cost AHRS [26]. It operated on a similar principal to the pitch and roll functions of an AHRS, but using gimbaled, instead of strapdown, sensors.

A spinning mass gyro with vertical spin axis is mounted in a pair of gimbals. When the host vehicle tilts, the gyro spin axis remains vertical and the roll and pitch are obtained from the gimbal readouts.

The spin axis will move relative to the vertical due to instrument errors and rotation of the horizontal plane with respect to inertial space. This is detected by an orthogonal pair of horizontal accelerometers mounted on the gyro platform. They are used to control torquers that maintain the gyro spin axis in the vertical direction. The accelerometers will sense vehicle acceleration as well as platform tilt, which can cause erroneous platform torquing. This is mitigated by time-smoothing the accelerometer signals, i.e. using a relatively low gain in the torquer control loops. The time constant is determined by how quickly the gyro drifts out of alignment. With poor quality sensors this can be as low as 30 seconds, which can be shorter than the time taken for the aircraft to maneuver. Also, as the accelerometer axes within the horizontal plane are aligned with the aircraft body, the accelerations sensed will not cancel over the maneuver, particularly in the across-track direction. Therefore, an upper limit on the signals from the accelerometers was often applied to limit the effects of vehicle acceleration on platform alignment.

C.7.4 Early Inertial Navigation Systems

Experimental INS were first developed in the 1920s. However, the sensor and computation technology of the time was not good enough for practical application. The first application of inertial navigation technology was in German V2 rockets during World War II. A partial INS, comprising 3 gyros and a single platform-mounted accelerometer, was used during the initial boost phase only to determine when the desired velocity had been reached and the rocket motor therefore should be shut down; it was not used for the full flight [7].

The first true INSs, capable of useful navigation for an hour or more, were developed in the 1950s. They were initially very large, expensive and restricted to military use. However, during the 1960s they were adopted by civil aviation for long-range navigation. In 1968, the Boeing 747 became the first airliner to be fitted with INS as standard and not equipped for astronomical navigation. Shorter range commercial aircraft did not use INS until the 1980s [7].

Early INS were all of the platform configuration, as described in Section 5.7 of [0] and in [27–29]. They used analog computation and, instead of applying temperature-dependent sensor calibration, were maintained at a stable temperature; this required a 30 minute warm-up period between power-up and alignment.

Strapdown INS were initially deployed in guided weapons where compactness was important and accurate angular rate measurements were needed for the guidance and control system. They replaced platform systems in new aircraft during the 1980s.

C.7.5 Stellar-Inertial Navigation

Prior to the advent of GNSS, stellar navigation was the only way of constraining the position error of an aircraft navigation system to less than 1 km at any location. Transit and Tsikada accuracy was severely degraded on high speed vehicles, Omega was not accurate enough and other radio navigation signals were not available globally. Thus stellar-inertial navigation systems found application in military reconnaissance aircraft and long-range bombers that remained in the air for many hours. Early systems combined a telescopic star tracker (see Section 11.4.1 of [0]) with a platform INS, such that the star tracker azimuth and elevation gimbals were mounted on the same platform as the inertial sensors. Strapdown star imagers only replaced gimbaled start trackers comparatively recently.

The star tracker constrains the INS attitude error, preventing the gyro biases from producing unbounded error growth. The effect of the remaining INS errors on horizontal positioning is then constrained by the Schuler feedback [30].

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Additional Acronyms

BABS	Blind-approach beacon system
DECTRA	Decca Track
DELRAC	Decca Long-Range Area Cover
LF	Low frequency
MLS	Microwave Landing System
MSBLS	Microwave Scanning Beam Landing System
POPI	Post Office Position Indicator
RDSS	Radio determination satellite service
RSDN	Normal long-range navigation system (Russian acronym)
SS	Skywave Synchronized
VAR	Visual Aural Range
VLF	Very low frequency

See [0] for the remaining acronyms.